MATH 110: Linear Algebra

Final Study Guide

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Instructions: You will be able to use any printed or written notes, but no internet, digital notes, or textbooks.

1. Midterm study terms $\oplus$ the following:
   - Eigenvalue
   - Eigenvector
   - Polynomials of operators
   - Upper triangular matrix, diagonal matrix
   - Inner product, properties
   - Euclidean inner product
   - Norm, basic properties
   - Orthogonal
   - Orthonormal
   - Orthogonal complement, basic properties
   - Orthogonal projection, basic properties
   - Adjoint
   - Self-adjoint operator
   - Normal operator
   - Positive operator
   - Square root
   - Isometries
   - Generalized eigenvector, generalized eigenspace
   - Nilpotent operator
   - Algebraic multiplicity
   - Geometric multiplicity
   - Block diagonal matrix
   - Characteristic polynomial
   - Minimal polynomial
   - Jordan basis
   - Trace, basis properties
   - Determinant, basis properties
2. Computations:
   - How to compute eigenvalues, eigenspaces
   - Computing the matrix of an operator
   - Computing generalized eigenspaces
   - Computations with characteristic and minimal polynomials

- Jordan Form for an operator

3. Key results and tools to review:
   - Equivalent conditions to be an eigenvector (Axler 5.6)
   - Linear independence of eigenvectors
   - Every operator on a complex vector space has an eigenvalue
   - Conditions for an upper triangular matrix
   - Over $\mathbb{C}$, every matrix has an upper-triangular form with respect to some basis
   - What does upper-triangular form tell us about invertibility, eigenvalues?
   - Conditions for diagonalizability
   - Pythagorean theorem
   - Orthogonal decomposition
   - Cauchy-Schwarz
   - Triangle-Inequality
   - Norm of a linear combination (Axler 6.25)
   - Writing a vector as a linear combination of an orthonormal basis (Axler 6.30)
   - Gram-Schmidt
   - Existence of orthonormal basis
   - Schur's theorem
   - Riesz Representation Theorem
   - $V = U \oplus U^{\perp}$
   - Matrix of T^* with respect to an orthonormal basis
   - Eigenvalues of self-adjoint operators are real
   - Normal if and only if $||Tv|| = ||T^*v||$
   - Complex and real spectral theorems
   - Characterizing positive operators
   - Characterizing isometries
   - Increasing sequence of null spaces and termination (Axler 8.2-8.4)
   - $V = \text{null } T^{\dim V} \oplus \text{range} T^{\dim V}$
   - $G(\lambda, T) = \text{null } (T - \lambda I)^{\dim V}$
   - Matrix of a nilpotent operator
   - Description of operators on complex vector spaces
   - Over $\mathbb{C}$, invertible operators have square roots
   - Cayley-Hamilton
   - Eigenvalues are zeros of minimal polynomial
   - Jordan Form exists for any $T \in \mathcal{L}(V)$ where V is complex