MATH 110: Linear Algebra

Homework 2

Instructor Franny Dean

Instructions: Please type your solutions to the following in LaTeX and upload your solutions to Gradescope by 4:10pm on **Wednesday, June 28, 2023**. You are highly encouraged to work with your classmates, but your write up must be done independently without looking at any other student's solutions.

1. (Axler 1.C.9) A function $f : \mathbb{R} \to \mathbb{R}$ is ***periodic*** if there exists a positive number p such that $f(x) = f(x+p)$ for all $x \in \mathbb{R}$. Is the set of periodic functions from $\mathbb{R}$ to $\mathbb{R}$ a subspace of $\mathbb{R}^{\mathbb{R}}$? Prove it.

2. (Axler 1.C.10) Suppose $U_1, U_2 \subseteq V$ as subspaces. Prove that the intersection $U_1 \cap U_2$ is a subspace.

3. (Axler 1.C.24) A function $f : \mathbb{R} \to \mathbb{R}$ is called ***even*** if $f(-x) = f(x)$ for all $x \in \mathbb{R}$. A function $f : \mathbb{R} \to \mathbb{R}$ is called ***odd*** if $f(-x) = -f(x)$ for all $x \in \mathbb{R}$. Let U_e denote the set of real-valued even functions and U_o the real-valued odd functions on $\mathbb{R}$. Show $\mathbb{R}^{\mathbb{R}} = U_e \oplus U_o$.

4. (Axler 2.A.3) Find a number t such that $(3,1,4), (2,-3,5), (5,9,-t)$ is not linearly independent in $\mathbb{R}^3$.

5. (Axler 2.A.11) Suppose $v_1, \ldots, v_m$ is linearly independent in V and $w \in V$. Show that $v_1, \ldots, v_m, w$ is linearly independent if and only if $w \notin \operatorname{span}(v_1, \ldots, v_m)$.