MATH 110: Linear Algebra

Homework 3

Instructor Franny Dean

Instructions: Please type your solutions to the following in LaTeX and upload your solutions to Gradescope by 4:10pm on **Wednesday, July 5, 2023**. You are highly encouraged to work with your classmates, but your write up must be done independently without looking at any other student's solutions.

1. (Axler 2.B.6) Suppose v_1, v_2, v_3, v_4 is a basis of V. Prove that $v_1 + v_2, v_2 + v_3, v_3 + v_4, v_4$ is also a basis of V.

2. (Axler 2.C.12) Suppose U and W are both five-dimensional subspaces of $\mathbb{R}^9$. Prove $U \cap W \neq \{0\}$.

3. (Axler 3.A.10) Suppose U is a subspace of V with $U \neq V$. Suppose $S \in \mathcal{L}(U, V)$ and $S \neq 0$ (which means $Su \neq 0$ for some $u \in U$). Define $T : V \to W$ by

$$Tv = \begin{cases} Sv & \text{if } v \in U \\ 0 & \text{if } v \in V \text{ but } v \notin U. \end{cases}$$

   Prove that T is not a linear map on V.

4. (Axler 3.A.4) Suppose $T \in \mathcal{L}(V, W)$ and $v_1, \ldots, v_m$ is a list of vectors in V such that $Tv_1, \ldots, Tv_m$ are linearly independent. Prove that $v_1, \ldots, v_m$ are linearly independent.

5. (Axler 3.B.8) Suppose V and W are finite-dimensional with $\dim V \geq \dim W \geq 2$. Show that

$$\{T \in \mathcal{L}(V, W) : T \text{ is not surjective } \}$$

   is not a subspace of $\mathcal{L}(V, W)$.