MATH 110: Linear Algebra

Homework 5

Instructor Franny Dean

Instructions: Please type your solutions to the following in LaTeX and upload your solutions to Gradescope by 4:10pm on **Wednesday, July 19, 2023**. You are highly encouraged to work with your classmates, but your write up must be done independently without looking at any other student's solutions.

1. Read Axler Chapter 4.
2. (Axler 3.F.13) Define $T : \mathbb{R}^3 \to \mathbb{R}^2$ by $T(x, y, z) = (4x + 5y + 6z, 7x + 8y + 9z)$. Suppose ϕ_1, ϕ_2 denotes the dual basis of the standard basis of $\mathbb{R}^2$ and ψ_1, ψ_2, ψ_3 denotes the dual basis of the standard basis of $\mathbb{R}^3$.
   (a) Describe the linear functionals $T'(\phi_1)$ and $T'(\phi_2)$.
   (b) Write $T'(\phi_1)$ and $T'(\phi_2)$ as linear combinations of ψ_1, ψ_2, ψ_3.
3. (Axler 5.A.1) Suppose $T \in \mathcal{L}(V)$ and U is a subspace of V.
   (a) Prove that if $U \subset \text{null } T$, then U is invariant under T.
   (b) Prove that if $\text{range } T \subset U$, then U is invariant under T.
4. (Axler 5.A.7) Suppose $T \in \mathcal{L}(\mathbb{R}^2)$ defined by $T(x, y) = (-3y, x)$. Find the eigenvalues of T.
5. (Axler 5.A.8) Define $T \in \mathcal{L}(\mathbb{F}^2)$ by $T(w, z) = (z, w)$. Find all eigenvalues and eigenvectors of T.
6. (Axler 5.A.20) Find all eigenvalues and eigenvectors of the backward shift operator $T \in \mathcal{L}(\mathbb{F}^\infty)$ defined by $T(z_1, z_2, z_3, \ldots) = (z_2, z_3, \ldots)$.